Learning Topology and Geometry

Automated Grammar Induction

Linas Vepstas

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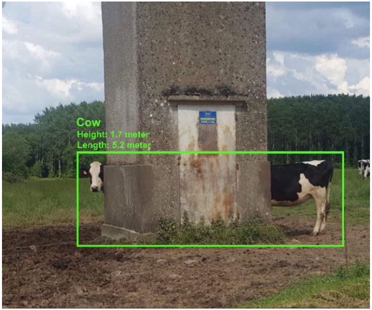
Learning Topology and Geometry

A Lack of Topological and Geometric Awareness

Critiques of DL/NN recently circulating on social media



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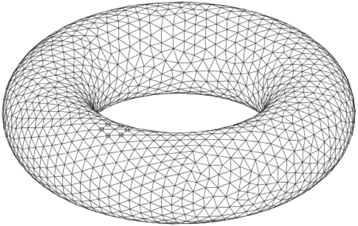
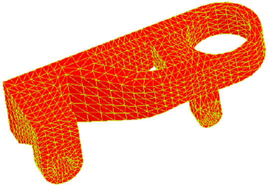
Conventional Simplical, Cellular Homology

Triangulations, cycles, cocycles, universal covering groups, metrics





Deep and broad mathemaical foundations to draw on.

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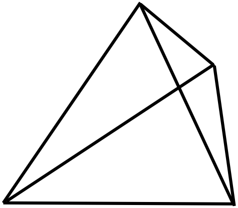
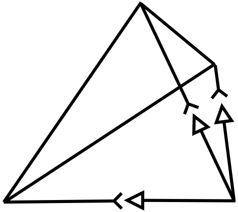
Reframe: Edge Lists -> Jigsaws with Connectors

Jigsaws, plus "global" constraints such as must-form-a-cycle





Not quite graph theory.
Not quite relational algebra.
... but almost so.

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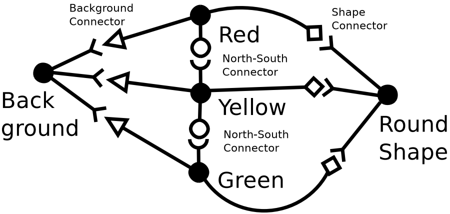
Connectors Indicate Symbolic Relationships

Image segmentation as labelled geometric relationships



Geometric syntax encodes part–whole relationships!

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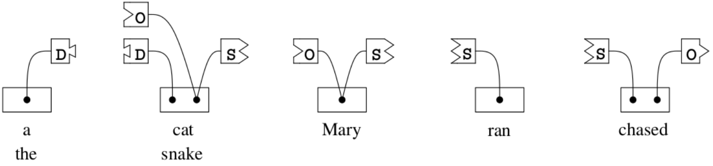
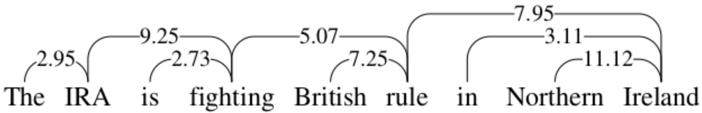
Jigsaw Paradigm Established in Linguistics

Syntax in Link Grammar (1991) and earlier (Marcus, 1967)





Maximum Spanning Tree parse from Word-Pair MI (1998)

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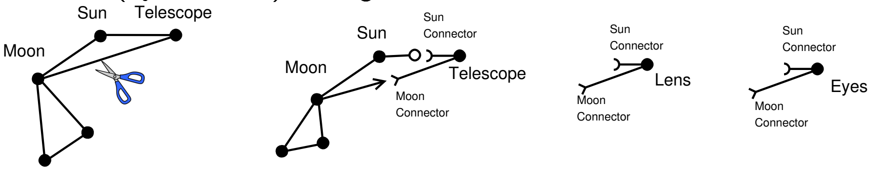
Provides Semantics for Symbolic AI

Abstract (Symbolic AI) "Things that can be seen"



Syntax extending into shallow semantics

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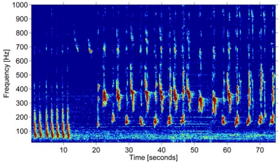
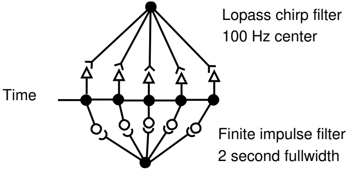
Not Just 1D, 2D, 3D, but also Abstract Sensory Domains

Audio: frequency, intensity, time, envelope, chirp modulation
More generally: wavelet-style decomposition





Syntax and structure of a whale song

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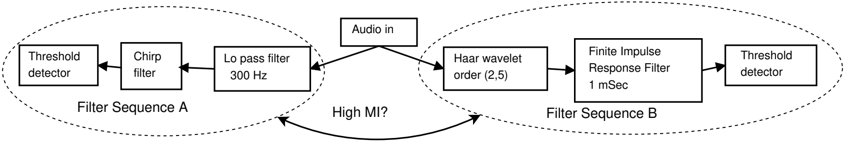
Segmentation and Tokenization as (evolutionary, ML) Program Learning

Conventional ML/AI can explore DSP filter sequences



Can DL/NN be used to generate these?
Possibly ... probably. Not been done.
Recursive... (model->syntax->model->syntax...)
... and deep ("cheap").

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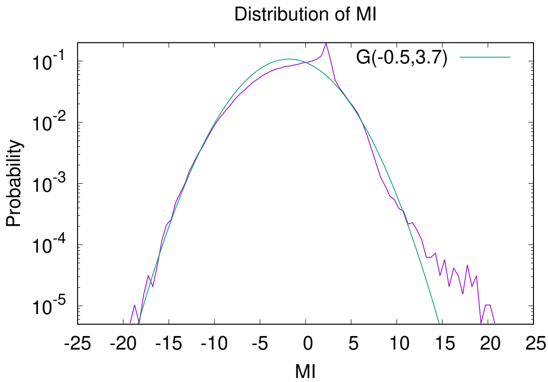
Experimental results

Gaussian Orthogonal Ensemble (Spin Glass)



Uniform distribution of English word similarities in high dimensions.
Conventional (information-theoretical) metrics apply.

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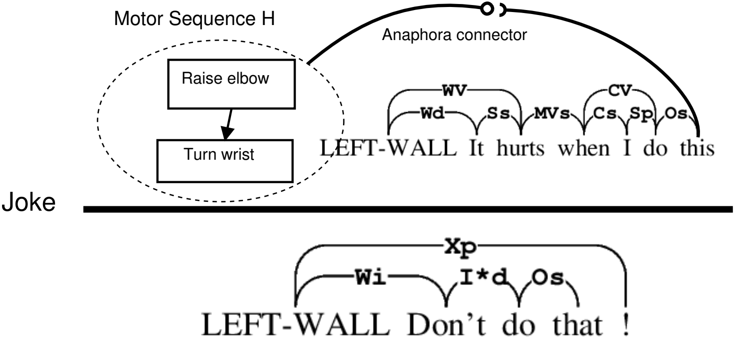
Common Sense as Inference over Symbolic Domains

- Enactive AI founded on unsupervised symbolic relationships.
- “Common Sense” can be learned recursively i.e. “deeply”.



- GOFAI failed because it depended on human-curated datasets.
- This proposal doesn’t, but it remains (mostly) symbolic.
- GOFAI was shallow. Shallow==hard-to-learn.

Automated Grammar Induction

Experimental Results

Linas Vepštas

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Basic definitions:

- Word Pair: (u, w)
- Count: $N(u, w)$
- Frequentist probability: $p(u, w) = N(u, w) / N(*, *)$
- Star == wildcard sum over all entries in that location
- Lexical Attraction (MI):

$$MI(u, w) = \log_2 \frac{p(u, w)}{p(u, *)\, p(*, w)}$$

- Not symmetric: $(u, w) \neq (w, u)$

Sparse matrix with global properties

- Log width and height: $\log_2 N_L$ and $\log_2 N_R$
- Log total number of nonzero entries: $\log_2 D_{\mathrm{Tot}}$
- Log total number of observations: $\log_2 N_{\mathrm{Tot}}$
- Sparsity: $-\log_2 D_{\mathrm{Tot}} / N_L \times N_R$
- Rarity: $\log_2 D_{\mathrm{Tot}} / \sqrt{N_L \times N_R}$ is independent of dataset size!
- Entropy: $H_{\mathrm{Tot}} = \sum_{w,v} p(w,v) \log_2 p(w,v)$
- Marginal Entropy: $H_{\mathrm{Left}} = \sum_{w} p(w,*) \log_2 p(w,*)$
- Total MI:

$$MI = H_{\mathrm{Tot}} - H_{\mathrm{Left}} - H_{\mathrm{Right}} = \sum_{w,v} p(w,v) \log_2 \frac{p(w,v)}{p(w,*)\, p(*,v)}$$

Experimental Results

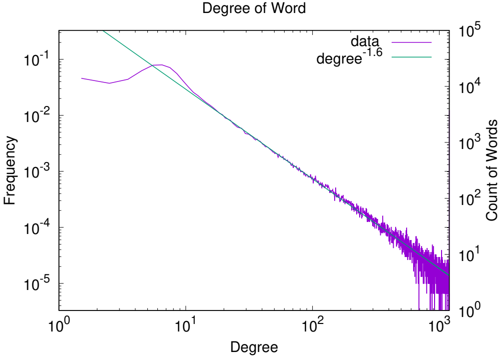
Example Word-Pair Data Sets

Corpus	1	2	3	4	5
$\log_2 N_L$	16.678	17.097	18.214	18.600	19.019
$\log_2 N_R$	16.690	17.117	18.228	18.620	19.039
$\log_2 D_{\mathrm{Tot}}$	23.224	23.797	24.748	25.180	25.627
Sparsity	10.144	10.416	11.693	12.040	12.431
Rarity	6.540	6.690	6.527	6.570	6.598
$\log_2 N_{\mathrm{Tot}}/D_{\mathrm{Tot}}$	4.779	5.079	5.128	5.235	5.335
Total Entropy	17.827	17.889	18.378	18.503	18.631
Left Entropy	9.7963	9.8102	10.069	10.109	10.148
Right Entropy	9.5884	9.5463	9.8321	9.8801	9.9265
MI	1.5572	1.4677	1.5227	1.4863	1.4431

Experimental Results

Sample Size Effects

Vertex degree: For word w, how many pairs (u, w) is it in?

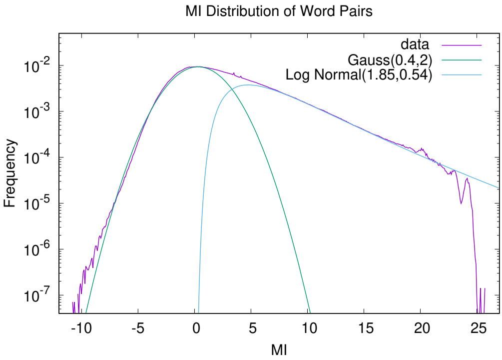


- Zipfian, with exponent $\gamma \approx 1.6$.
- Left side: 2/3rds of the data-set contains junk: bad punctuation, typos, bad quote segmentation, stray markup.

Experimental Results

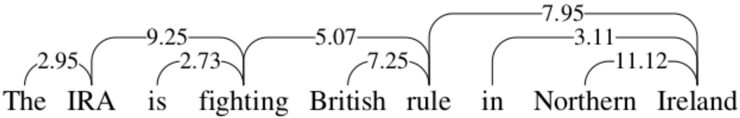
MI Distribution

28 Million word-pairs



- Sum of two curves: Gaussian and Log–Normal
- Theory: ??? Gaussian is presumably "common-mode noise"
- Uniform random under-sampling of pairs -> Gaussian
- Same for Mandarin Chinese

Maximum Spanning Tree Parse of English.



- Cutting each edge in half yields jigsaws ("disjuncts")
- Count these – Count word-jigsaw pairs (w, d)
- Repeat the matrix game.
- Matrix is (very) rectangular

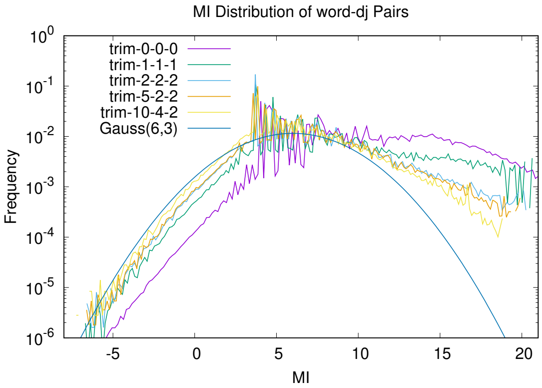
Experimental Results

Jigsaw Data Sets Characterization.

Trim cuts	full set	1-1-1	2-2-2	5-2-2	10-4-2
$\log_2 N_{\text{words}}$	18.526	15.542	13.644	12.889	12.249
$\log_2 N_{\text{disjuncts}}$	24.615	20.599	18.662	18.447	17.369
$\log_2 D_{\text{Tot}}$	24.761	20.967	19.247	19.086	18.443
Sparsity	18.380	15.174	13.058	12.251	11.175
Rarity	3.191	2.896	3.095	3.418	3.634
$\log_2 N_{\text{Tot}}/D_{\text{Tot}}$	0.356	2.248	3.384	3.461	3.889
Total Entropy	24.100	19.486	17.711	17.508	16.875
Left Entropy	23.494	18.346	16.417	16.163	15.379
Right Entropy	10.157	7.937	7.280	7.268	7.258
MI	9.550	6.796	5.987	5.923	5.763

Experimental Results

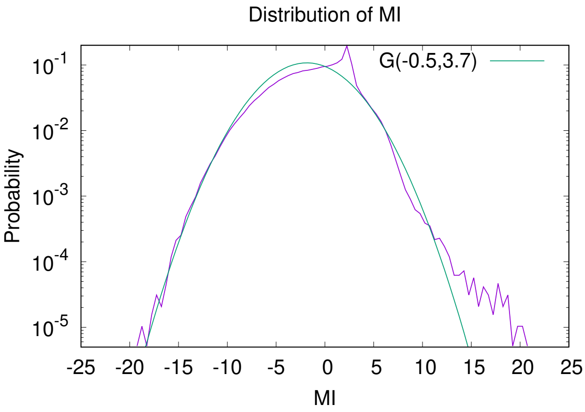
Distribution of Jigsaw (Disjunct) MI



- This is $MI(w, d)$ for word w and jigsaw d
- Unclean. Obscure meaning.

Experimental Results

Distribution of Similarity



- Wow! Gaussian!

Experimental Results

Similarity Metrics

- Inner product: $i(w,v) = \sum_d p(w,d)\, p(v,d)$
- MI of inner product:

$$MI(w,v) = \log_2 \frac{i(w,v)\, i(*,*)}{i(w,*)\, i(v,*)}$$

- Variation of Information (VI):

$$VI(w,v) = \log_2 \frac{i(w,v)}{\sqrt{i(w,*)\, i(v,*)}}$$

- Various Jacquard distances...
- *Not the cosine distance!!! Its terrible!*

Gaussian Orthogonal Ensemble

- A high–dimensional sphere.
- With a uniform random distribution on it.
- Dimension of space == size of vocabulary.
- A vector for word w has direction $MI(w, u)$.
- Each vector corresponds to the syntactic usage of that word.
- Syntax is maximally leveraged by English speakers!
- Probably holds in other languages, too.
- This is about the effectiveness of grammar in communications.

Experimental Results

Similarity and Clustering

Clustering generalizes from specifics

Top–ranked Clusters		
+ — “ ” _	? . !	must would
, ;	He It I There	he she
was is	of in to from	are were
but and that as	has was is had could	might should will may

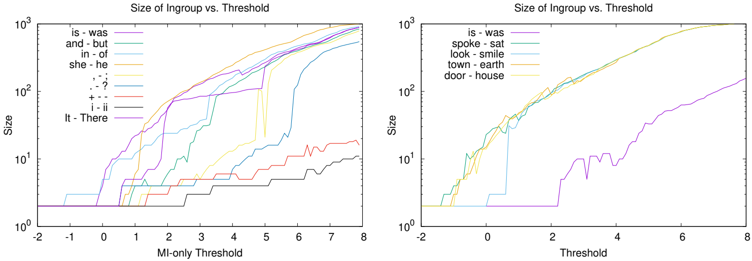
Not “just” similar words, but also:

- Similar grammatical behavior.
- Similar structure.
- Similar semantics.

Experimental Results

Word-sense Disambiguation

Each word–vector is a linear sum of multiple word-senses



- Exclusive club, Common interests
- How exclusive?
  - There's a natural threshold to nearest neighbors.
- Common interests?
  - Disjuncts not shared by majority are different word senses

Experimental Results

Conclusion

We've learned:

- Information–theoretic foundations are central.
- Experimental confirmation is central.
- Structure can be extracted from undifferentiated samples.
- Structure is a synonym for grammar.
- Recursion: structure defines a new random, uniform sampling.
- So sample again, to find differences and structure at the next level.